Propagation Characteristics of Aberrated Partially Coherent Flat-topped Beam in Turbulent Atmosphere

F. D. Kashani and B. Ghafary
F_dk@iust.ac.ir, ghafary@iust.ac.ir
Iran University of Science and Technology, Tehran, Iran

Abstract

In this paper, we investigate the characteristics of a partially coherent flat-topped (PCFT) beam through turbulent atmosphere. The analytical expressions for the spectral density and the beam width of the astigmatic PCFT beams with circular symmetry propagating in turbulent atmosphere are derived. It is shown that the spectral density and beam width of these beams in turbulent atmosphere are heavily influenced by the astigmatism coefficient, as well as by the order of flatness. Also, analytical formulas for beam width of aberrated PCFT beams propagated through turbulent atmosphere are derived, which show that higher order aberrated PCFT beams are more spread, but aberrated PCFT beams are not affected by correlation length. We also compute the effective width of the spectral degree of coherence for aberrated PCFT beams, which evidently show that aberrated beams with higher astigmatism coefficient are more influenced in turbulent atmosphere.

Introduction

Propagation of the laser beams through random media has attracted much attention due to their applications in free space optical communications [1]. It is shown that partially coherent beams are less influenced by turbulent atmosphere than completely coherent beams [2]. Therefore, a considerable number of investigators have paid attention to the characterizations of partially coherent light propagating through turbulent atmosphere [3–12]. Because of less effectiveness of PCFT beams in turbulent atmosphere, partially coherent flat-topped (PCFT) beams have been widely studied [13]. For characterizing PCFT beams, it is necessary to investigate the spectral density, the spectral degree of coherence (SDC), and the spectral degree of polarization (SDP) that may change on propagating in turbulent atmosphere. Some investigators study these characteristics in turbulent atmosphere [14]. However, most of the publications on this subject were devoted to the aberration-free case.

On the other hand, it is more appropriate to introduce the aberrations as phase functions that modify the wavefront for the propagation of a laser beam [15, 16]. As is well known, the aberrations may emerge while the laser beam passes through an optical system. In this sense, it is worth studying the propagation characteristics of the beams with aberrations. An aberration may be classified as defocus, astigmatism, spherical aberration, etc. According to the investigations concerning the aberrations, the size and the shape of the beam were strongly influenced by the aberrations [1, 14, 17, 18, 19, 20, 21, 22]. Chen and Pu [1] studied
the propagation characteristics of stochastic electromagnetic beams with astigmatic aberration in turbulent atmosphere; however, to the best of our knowledge, there are no papers dealing with the influence of the aberration on the propagation characteristics of PCFT beams in a turbulent atmosphere.

In this paper, we investigate the characteristics of PCFT beam with astigmatic aberration propagating through turbulent atmosphere. Taking aberrated PCFT beams with circular symmetry, the analytical expressions for the intensity and SDC of the beams propagating through turbulent atmosphere are derived.

It is shown that the spectral density of the PCFT beams are influenced by the astigmatism coefficients, as well as SDC, and the effective width of the spectral degree of coherence (EWSDC). It is shown that astigmatic PCFT beams are influenced by correlation property, order of flatness, and the atmospheric turbulence, as well as aberration-free ones. It is shown analytically that with a fixed set of source parameters and under a particular atmospheric turbulence model, a PCFT beam propagating through atmospheric turbulence reaches its maximum value of coherence after it propagates a particular distance, and the EWSDC also has its maximum value. The astigmatism coefficients affect the maximum value of the EWSDC and distance that occurs.

**Propagation of PCFT Beams with Astigmatic Aberrations in a Turbulent Atmosphere**

We consider a field propagation from the plane Z=0 (i.e., the partially coherent source plane) into the half space Z>0 where the turbulence exists. Figure 1 illustrates the notation relating to the propagation of mutual coherence function (MCF). The second-order correlation properties of such a source, at two points \((\tilde{\rho}_1, z)\) and \((\tilde{\rho}_2, z)\) in transverse plane \(Z=\text{const}>0\), may be characterized by a \(2\times 2\) cross-spectral matrix [1, 14],

\[
\hat{W} (\tilde{\rho}_1, \tilde{\rho}_2, z; \omega) = W_{ij} (\tilde{\rho}_1, \tilde{\rho}_2, z; \omega) = \left\langle E^*_i (\tilde{\rho}_1, z; \omega) E_j (\tilde{\rho}_2, z; \omega) \right\rangle
\]

where \(E_i (\tilde{\rho}, z; \omega)\) and \(E_j (\tilde{\rho}, z; \omega)\) are components of the complex electric vector at a point specified by transverse position vector \(\tilde{\rho}\), the asterisk stands for the complex conjugate. Angle brackets represent the average, taken over an ensemble of realizations of the electric field in the sense of the coherence theory in the space-frequency domain.
The elements of the cross-spectral density matrix in the source plane can be expressed by:

\[ W_{q}^{(s)}(\tilde{\rho}'_1, \tilde{\rho}'_2, 0; \omega) = \langle E_{q}^{(s)}(\tilde{\rho}'_1; \omega) E_{q}^{(s)}(\tilde{\rho}'_2; \omega) \rangle \]  

(2)

The elements of the cross-spectral density matrix at the two points \((\tilde{\rho}_1, z)\) and \((\tilde{\rho}_2, z)\) in transverse plane \(Z=\text{const} >0\) are given by [1],

\[
W_{s}(\tilde{\rho}_1, \tilde{\rho}_2, z; \omega) = \left(\frac{k}{2\pi z}\right) \int d^2\rho' \times \int d^2\rho'' W_{s}^{(s)}(\tilde{\rho}'_1, \tilde{\rho}'_2, 0; \omega) \\
\times \exp\left[-ik\left((\tilde{\rho}_1 - \tilde{\rho}_1') - (\tilde{\rho}_2 - \tilde{\rho}_2')\right)\right] \times \left\langle \exp\left(\psi'(\tilde{\rho}_1, \tilde{\rho}_1', z; \omega) + \psi(\tilde{\rho}_2, \tilde{\rho}_2', z; \omega)\right) \right\rangle_m \\
\times K(\tilde{\rho}'_1, \tilde{\rho}'_2) 
\]  

(3)

where \(\langle \ldots \rangle_m\) denotes averaging over the ensemble of statistical realizations of the atmospheric turbulence. It is assumed here that the fluctuation of the light beam and of the turbulent atmosphere is irrelevant. As the aberration is taken into consideration here, the aberration function should be added as a propagation factor [1, 20]. It is worth mentioning that aberration can be created through an optical device or atmosphere. Or, it could be inherent like astigmatism in laser diode; then, the propagator \(K\) is given in the following formula,

\[ K(\tilde{\rho}_1, \tilde{\rho}_2) = \exp(ik[\phi(\tilde{\rho}_1') - \phi(\tilde{\rho}_2')]) \]  

(4)

where \(\phi\) is the wave aberration function. In equation (3) the angular bracket that describes the turbulence effect can be approximated as

\[
\langle \exp\left(\psi'(\tilde{\rho}_1, \tilde{\rho}_1', z; \omega) + \psi(\tilde{\rho}_2, \tilde{\rho}_2', z; \omega)\right) \rangle_m \approx \\
\exp\left[\frac{-\pi k^2 z}{3} \left(\rho_1^2 + (\rho_1 - \rho_2)^2 + (\rho_1' - \rho_2')^2 + (\rho_1'' - \rho_2'')^2\right)\right] \int k^2 \phi_k(\kappa) d\kappa' 
\]  

(5)
where the quantity \( \int \kappa \phi_\kappa (\kappa) d\kappa \) describes the effect of turbulence, \( \phi_\kappa (\kappa) \) being the spectrum of the refractive-index fluctuations that can be characterized by the Tatarskii model or the Kolmogrov model [21]. Figure 2 shows a system diagram of aberrated beam propagation.

![Figure 2: System Diagram of Aberrated Beam Propagation](image)

As for the PCFT source, \( W_{ij}^{(0)}(\tilde{\rho}_i', \tilde{\rho}_j'; 0; \omega) \) can be written as

\[
E_{\omega} (\tilde{\rho}_i'; \omega) = \sum_{n=1}^{N} A_n \frac{(-1)^{n-1}}{N} \binom{N}{n} \exp \left( -\frac{n\tilde{\rho}_i'^2}{2\sigma_i^2} \right)
\]

\[
g^{(s)}(\tilde{\rho}_i'-\tilde{\rho}_j'; \omega) = \sum_{n=1} B_n \exp \left( -\frac{c(\tilde{\rho}_i'-\tilde{\rho}_j')^2}{2\sigma_{ij}^2} \right)
\]

\[
W_{ij}^{(s)}(\tilde{\rho}_i', \tilde{\rho}_j'; 0; \omega) = \sum_{c=1}^{N} \sum_{n=1}^{N} \sum_{m=1}^{N} A_i A_j B_{ij} \frac{(-1)^{n+m}}{N^2} \binom{N}{n} \binom{N}{m} \exp \left( -\frac{(n\tilde{\rho}_i'^2 + m\tilde{\rho}_j'^2)}{2\sigma_i^2} \right)
\]

\[
\times \exp \left( -\frac{c(\tilde{\rho}_i'-\tilde{\rho}_j')^2}{2\sigma_{ij}^2} \right)
\]

where the coefficients \( A_i, B_j, \sigma_s \), and \( \sigma_g \) are positive quantities [21–23], which are independent of position. The coefficients \( B_{ij} \) satisfies the relations

\[
B_{ij} = 1, \quad \text{when } i \neq j \quad (7a)
\]

\[
|B_{ij}| \leq 1, \quad \text{when } i = j
\]

and

\[
B_{ji} = B_{ij}^* \quad (7b)
\]

The parameters \( \sigma_s \) and \( \sigma_g \) characterize the effective source size and the effective width of the spectral degree of coherence of source, respectively. Parameters characterizing an
electromagnetic Gaussian source cannot be chosen arbitrarily due to the sufficient conditions of which parameters the source must satisfy. The restrictions on the choice of parameters of an electromagnetic Gaussian source are provided in references [22, 24]. These restrictions are imposed in this paper. The wave aberration function for astigmatism is characterized by

\[ \varphi = c_a (x^2 - y^2) \]  

(8)

where \( c_a \) is the astigmatism coefficient, and other aberrations can be neglected.

Substituting equation (8) into equation (4), then substituting equations (4), (5), and (6c) into equation (3), and calculating the related integral \( W_\omega (\tilde{\rho}_1, \tilde{\rho}_2, z; \omega) \) is obtained as

\[
W_\omega (\tilde{\rho}_1, \tilde{\rho}_2, z; \omega) = \frac{k^2}{z^2} \sum_{i=1}^{N} \sum_{j=1}^{N} A_i A_j \frac{(-1)^{i+j}}{N^2} \left( \begin{array}{c} N \\ n \\ m \end{array} \right) \frac{1}{4\alpha_i \alpha_j - \alpha_i' \alpha_j'} \times \exp \left( \frac{\alpha_i^2 \alpha_j^2 - \alpha_i' \alpha_j'}{4\alpha_i \alpha_j - \alpha_i' \alpha_j'} \right) \]

(9)

where

\[
\begin{align*}
\alpha_i &= \frac{n + m}{4\sigma_i^2} - \frac{c}{2\sigma_i^2} + M, \\
\alpha_j &= \frac{n + m}{4\sigma_j^2} - \frac{c}{2\sigma_j^2} + M, \\
\alpha_3 &= \frac{-ik}{2z} (\rho_{1s} - \rho_{2s}), \\
\alpha_4 &= \frac{-ik}{2z} (\rho_{1s} + \rho_{2s}), \\
\beta_i &= \frac{-ik}{z} (\rho_{1s} - \rho_{2s}), \\
\beta_j &= \frac{-ik}{z} (\rho_{1s} + \rho_{2s}) - M (\tilde{\rho}_1 - \tilde{\rho}_2), \\
\eta &= \exp \left( \frac{-ik}{2z} (\rho_1^2 - \rho_2^2) - M (\tilde{\rho}_1 - \tilde{\rho}_2) \right)
\end{align*}
\]

and the quantity \( M \) can represent as \( 0.5465C_n^{\frac{3}{5}}kz \) for the Tatarskii spectrum and as \( 0.49(C_n^{\frac{3}{5}}kz)^{\frac{3}{5}} \) for the Kolmogrov spectrum [22], with \( C_n^{\frac{3}{5}} \) being the refractive index structure parameter and \( l_0 \) being the inner scale of turbulence.

Expression (9) is the main result of this paper, by which one can study the change in degree of coherence for the partially coherent beam propagating through the atmospheric turbulence. According to the unified theory of coherence and polarization [14], the degree of coherence can be given as

\[
\mu (\tilde{\rho}_1, \tilde{\rho}_2, z; \omega) = \frac{Tr W (\tilde{\rho}_1, \tilde{\rho}_2, z; \omega)}{\sqrt{Tr W (\tilde{\rho}_1, \tilde{\rho}_1, z; \omega) Tr W (\tilde{\rho}_2, \tilde{\rho}_2, z; \omega)}} = \frac{W_{xx} (\tilde{\rho}_1, \tilde{\rho}_1, z; \omega) + W_{yy} (\tilde{\rho}_1, \tilde{\rho}_1, z; \omega)}{\sqrt{W_{xx} (\tilde{\rho}_1, \tilde{\rho}_1, z; \omega) + W_{yy} (\tilde{\rho}_1, \tilde{\rho}_1, z; \omega)}}
\]

(10)

where \( (Tr) \) denotes the trace of the cross-spectral matrix. Equations (9) and (10) are the base of the results presented in the next section.
Discussion

The spectral density of aberrated PCFT beam can be calculated with $\hat{\rho}_r = \hat{\rho}_z = \hat{\rho}$:

$$I (\hat{\rho}, z ; \omega) = Tr \hat{W} (\hat{\rho}, \hat{\rho}, z ; \omega)$$  \hfill (11)

In Figure 3, the spectral density in the cross-plane ($Z=1m$) of the PCFT beam propagating through turbulent atmosphere for different values of astigmatism are plotted. In Figure 3, changes of spectral densities for different values of $c_s$ are shown. It is shown in Figure 3a that for the aberration-free case, the beam has a circular shape; yet, for the astigmatic case, the beam is no longer of circular symmetry but is elliptical.

![Figure 3: The spectral density in the plane Z=1 m for different values of the astigmatism: (a) $c_s = 0 m^{-1}$, (b) $c_s = 0.05 m^{-1}$, (c) $c_s = 0.1 m^{-1}$, (d) $c_s = 0.5 m^{-1}$, (e) $c_s = 1 m^{-1}$, (f) $c_s = 5 m^{-1}$.](image)

The other parameters are chosen as:

$$\lambda = 632.8 \, nm, \sigma_0 = 0.02 \, m, \, c_s = 1 \times 10^{-14} \, m^{2/3}, \sigma_z = 0.05 m, N = 4$$

In Figure 4a, the normalized spectral density of astigmatic PCFT beams are plotted versus “X,” the distance from the center of the beam for different values of the astigmatism coefficient. Figure 4b shows the normalized spectral density of astigmatic PCFT beams versus “Y.” All of the parameters are the same as for Figure 4a.
Figure 4a: The normalized spectral density of astigmatic PCFT beams are plotted versus “X.” All of parameters are the same as for Figure 3. Figure 4b: The normalized spectral density of astigmatic PCFT beams versus “Y.”
In Figure 5, the spectral density for a long propagation distance (Z=10 Km) is presented.

![Figure 5: Spectral density in the plane Z=10 Km for different values of the astigmatism.](image)

(a) $c_x = 0 \text{ m}^{-1}$, (b) $c_x = 0.001 \text{ m}^{-1}$, (c) $c_x = 0.002 \text{ m}^{-1}$, (d) $c_x = 0.005 \text{ m}^{-1}$. The other parameters are the same as for Figure 3.

The order of flatness’ effect is shown in Figure 6. It is evident that the increasing order of flatness causes increased beam spread. Correlation length does not affect spectral density.
Figure 6: The spectral density in the plane Z=10 Km for different order of flatnesses: astigmatism, (a) $N = 1$, (b) $N = 2$, (c) $N = 4$, (d) $N = 8$. The other parameters are chosen as $\lambda = 632.8\, \text{nm}$, $\sigma_0 = 0.02\, \text{m}$, $C_{w} = 1 \times 10^{-14}\, \text{m}^{-2/3}$, $\sigma_r = 0.05\, \text{m}$, $c_z = 0.005\, \text{m}^{-1}$

To investigate the beam spread, the beam width can be calculated by the first- and second-order moment of the squared modulus for the amplitude distribution. Therefore, the beam width in x- and y-directions are defined as [4]:

$$w_x = \frac{2 \iint x^2 \langle I(\rho, z, w) \rangle \, dx \, dy}{\iint \langle I(\rho, z, w) \rangle \, dx \, dy}$$

$$w_y = \frac{2 \iint y^2 \langle I(\rho, z, w) \rangle \, dx \, dy}{\iint \langle I(\rho, z, w) \rangle \, dx \, dy}$$

(12)

Substituting equation (11) into equation (12) and calculating the related integral, one can acquire following relation for beam width:
It can be found that the larger the astigmatism coefficient, the larger the spread (see Figure 7). Moreover, the difference between beam width on the x- and y-direction is negligible for a short propagation distance, but it cannot be neglected for a long propagation distance.

\[
\begin{align*}
w_x^2 &= \frac{2}{k^2} \sum_{m=1}^{N} \sum_{n=1}^{N} (-1)^{m+n} \binom{N}{m} \binom{N}{n} \frac{4a_x - a_y^2}{a_x^2} \\
w_y^2 &= \frac{2}{k^2} \sum_{m=1}^{N} \sum_{n=1}^{N} (-1)^{m+n} \binom{N}{m} \binom{N}{n} \frac{4a_y - a_x^2}{a_y^2}
\end{align*}
\]

(13)

Figure 7: The beam width in x- and y-direction as a function of propagation distance for different values of the astigmatism coefficients. N=4, the other parameters are the same as for Figure 6.

As shown in Figure 8, increasing order of flatness causes increased beam spread.
Figure 8: The beam width in x- and y-direction as a function of propagation distance for different values of flatnesses. $c_a = 0.002 m^{-1}$, the other parameters are the same as for Figure 7.

In the following, we will investigate the behavior of the degree of coherence. We conduct this investigation by calculating the effective width of the spectral degree of coherence. We define the width of the spectral degree of coherence $\overline{\rho}_z (z)$ of the beam in turbulence as that
separation $|\tilde{\rho}_i - \tilde{\rho}_j|$ of points in a transverse cross-section at which $|\mu(\tilde{\rho}_i, \tilde{\rho}_j, z; \omega)|$ drops from its maximum value unit (for $|\tilde{\rho}_i - \tilde{\rho}_j| = 0$) to the value $\frac{1}{e}$ [22].

At first, $\tilde{\rho}_\mu(z)$ increases but decreases after arriving at a maximum value. This behavior is completely explained in reference [22]. The strength of turbulence and the source parameters determine this behavior.

Using three different values of astigmatism coefficient, we study the influence of source parameters on the behavior of the spectral degree of coherence, as well as the effective width of the spectral degree of coherence of the beam propagating in atmospheric turbulence characterized by Tatarskii spectrum, as shown in Figure 9.

![Figure 9: The effective width of spectral degree of coherence as a function of propagation distance for different values of the astigmatism. Solid curve: $c_s = 0 \text{ m}^{-1}$; long-dashed curve: $c_s = 0.001 \text{ m}^{-1}$; short-dashed line: $c_s = 0.002 \text{ m}^{-1}$; $\sigma_\rho = 0.001 \text{ m}$; $N = 4$, the other parameters are the same as for Figure 6.](image)

**Conclusion**

In this article, we have derived an analytical formula for the spectral density, beam width, and the effective width of spectral degree of coherence for astigmatic PCFT beams. Source parameters have their influences on the behavior of beams. Here, we investigate the effects of astigmatism coefficient in the near and far field. It is found that the size and the shape of the beam were strongly influenced by the aberrations. The other analytical formulas for the width of these beams clarify that the beam with higher order of flatness and astigmatic...
coefficient are more spread. We also numerically analyzed the effective width of the spectral degree of coherence for various astigmatism coefficients, which demonstrate the increasing $c_a$ effect on the maximum value of the spectral degree of coherence. Our analysis results may find solutions to problems involving optical imaging, as well as atmospheric optical communication systems.

References


**Biography**

FATEMEH DABBAGH KASHANI was born in Tehran, Iran, on September 1981. She graduated from Iran University of Science and Technology (IUST) in 2004 and 2006 with a B.S. and an M.S., respectively, in physics. She is Ph.D. candidate of physics at IUST. Her current research interests include optical communication systems, wave propagation in turbulent atmosphere, and investigation of wavefront characteristics.

BIJAN GHAFARY was born in Tehran, Iran, on September 1962. He graduated from Tabriz University in 1986 and 1990 with a B.S. and an M.S., respectively, in physics. He graduated from Ritsumakan University in photonics on April 2000. Since 2000, he has been an associate professor in the physics department at Iran University of Science and Technology (IUST). His current research interests include optical communication systems, wave propagation in turbulent atmosphere, nonlinear optics, and photonic devices.